Coefficient of thermal expansion of metals by Fizeau's interferometer

Objective

To determine the coefficient of thermal expansion (α) for a brass rod using Fizeau's interferometer

Theory

Thermal expansion is a simple and universal property of material which is quite informative. It originates from the thermal vibrations of atoms of the material. Linear thermal expansion may be considered as the relative displacement of two points on a material due to absorption of thermal energy. The coefficient of linear expansion,α, of a material is defined as The coefficient of linear expansion (α) of a material at a given temperature, is defined as the fractional change in any dimension per unit change of temperature and expressed as follows:

$$\alpha = \frac{1}{L}\frac{\Delta L}{\Delta T} \qquad (1)$$

where L is the original length of the material and ΔL is the change in length for a change in temperature ΔT. The value of α varies with temperature and ranges widely from about $5\mathrm{x}10^{-6}$ to $50\mathrm{x}10^{-6}$ / ^{0}K for different materials.

To determine α, one needs to measure L, ΔL and ΔT as shown in Eq. 1. Usually measurement of L poses no problem. Let us consider the value of ΔL for a change in temperature by 20^0K for a material of length 1 cm. Assuming maximum value of α = $50\mathrm{x}10^{-6}$ / ^{0}K, ΔL is calculated as 10^{-3}cm. This is not an easily measurable length. Hence special arrangement (usually optical) is needed to measure such length changes.

A commonly employed method is based on the principle of interference. When rays of light from a monochromatic source fall on a wedge shaped air film, an interference fringe pattern is obtained. These fringes are named after French Physicist Armand Hippolyte Louis Fizeau (1819-1896), who used the interference of light to measure the dilation of crystals. The set up he used is known as Fizeau's interferometer. In this set up, a wedge shaped air film is formed between the surfaces of two inclined glass plates. When the film is by illuminated by light of wavelength λ at normal incidence, interference occurs between the light rays reflected from the surfaces of upper and lower plates and a fringe pattern consisting of hyperbolic lines with equal spacing is observed. Optical path difference between the direct and the reflected ray of light is given by,

$$\Delta = 2t + \lambda/2 \qquad (2)$$

where t is the thickness of the air-film enclosed between the glass plates at the point of interest. It is to be noted that the air-film has a variable thickness and each fringe corresponds to particular thickness. Hence, these fringes are known as *fringes of equal thickness*. The factor $\lambda/2$ takes into account the abrupt phase change of π radians suffered by the wave reflected from the top of glass plate P_2. We know the following from the interference phenomenon

$$\text{Condition for maxima: } \Delta = m\lambda \qquad (3)$$

$$\text{Condition for minima: } \Delta = (2m + 1)\lambda/2 \qquad (4)$$

where m = 0, ±1, ±2, …

Therefore, dark fringes (minima) would satisfy the following relation:

$$2t = m\lambda \qquad (5)$$

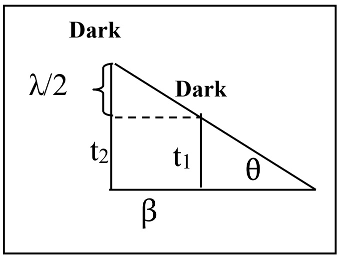

Fig. 1: angle of wedge shaped air film

Let the air film has thickness t_1 and t_2 for two consecutive dark fringes. Then the change in thickness, t_2-t_1, from one minimum to the next is $\lambda/2$. So, the angle of the air wedge (θ) can be expressed as (see Fig. 1)

$$\tan\theta = \lambda/2\beta \qquad (6)$$

where β is the fringe width, i.e. distance between two consecutive minima (or maxima).

If a test sample is introduced in between the two glass plates, then the wedge angle changes as the sample expands upon heating. One can estimate the change in length of the sample from the change in fringe spacing. The expansion in length of the rod (ΔL) increases the air-film thickness which, in turn, leads to an increase in the wedge angle ‘θ’. This geometry is represented in Fig. 2. Let thickness of the air film at point F be t_1 and t_2 at temperatures T_1 and T_2 (= T_1+ΔT), respectively. If the corresponding fringe widths are measured to be β_1 and β_2, then the angles of the wedges can be calculated respectively as follows

$$\theta_1 = \tan^{-1} (\lambda/2\beta_1)$$

$$\theta_2 = \tan^{-1} (\lambda/2\beta_2) \qquad (7)$$

Thus ΔL can be calculated as

$$\Delta L = l\Delta\theta = l(\theta_2 - \theta_1) \qquad (8)$$

where 'l' is the length of the glass plate 'AC'.

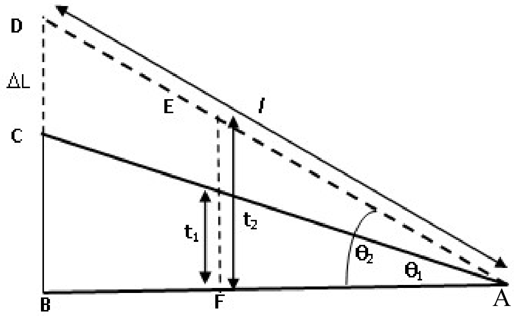

Fig. 2: Geometry to calculate the optical path difference

Finally, by using Eqs. (6-8)

$$\alpha = \frac{l\Delta\theta}{L_{RT}\Delta T} \equiv \frac{l}{L_{RT}}\left(\frac{\Delta\theta}{\Delta T}\right) \qquad (9)$$

The value of $\left(\frac{\Delta\theta}{\Delta T}\right)$ can be found by plotting a graph for $\theta \sim T$.

Typical values of α for aluminium, copper and brass at room temperature are 23.1×10^{-6}, $16.6 \times 10^{-6}\ K^{-1}$ and $20.3 \times 10^{-6}\ K^{-1}$, respectively

Apparatus:

- Two Glass plates
- Interferometer assembly
- Thermocouple and temperature indicator
- Travelling Microscope
- Variable transformer (variac)
- Sodium vapour lamp
- Specimen
- Heater

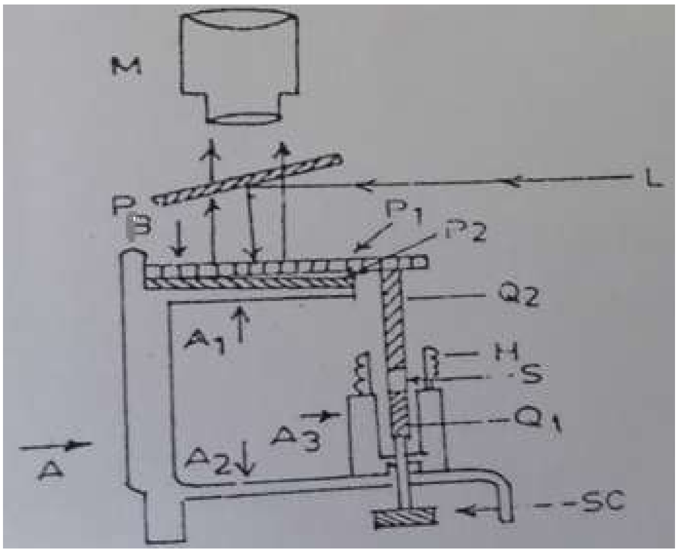

Fig. 3: Schematics of the set up

Description of the set up:

The schematic design of the experimental set up is shown in Fig. 3. It consists of an aluminium or brass stand A of about 15cm in height with two projections A_1 and A_2. A_1 holds two glass plates (microscopic slides) P_1 and P_2 where P_2 is slightly shorter than P_1. A screw SC passes up through A_2. Q_1 and Q_2 are two fused quartz rods holding the sample S between them. The sample is usually a small metal rod. A_3 is a metal block screwed on to A_2. At the top A_3 is supports a heater H which is a small cement tube over which a heating element is wound. A thermocouple (not shown) is inserted such that it is close to the sample. L is the source of light. The inclined glass plate P_3 reflects the light from L to the interferometer and transmits the light from the interferometer to the microscope. The actual set up and the observed fringes are shown in Fig. 4.

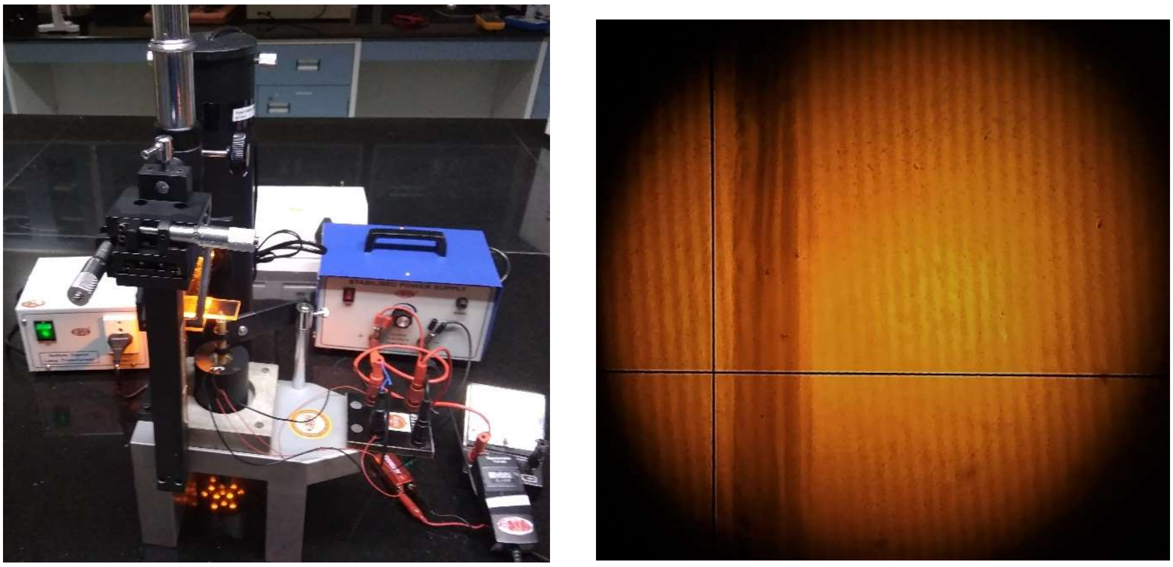

Fig. 4: Experimental set up and the Fizeau's fringes

Procedure

1. Record the initial temperature (room temperature) before beginning the experiment.
2. Measure length of the metal rod (L_{RT}) at room temperature using vernier calipers.
3. Place the sample rod between Q_1 and Q_2 such that Q_2 touches P_1 (Refer Fig. 3)
4. Switch on the sodium lamp and adjust P_3 so that it reflects light towards P_2. Looking through the microscope gently adjust the screw SC such that an air wedge is formed between P_1 and P_2 and a fringe pattern is observed.
5. Ensure that Q_2 touches P_1 properly. This can be done by rotating the screw (SC) below the platform in either direction and checking that the fringe spacing changes accordingly.
6. Carefully adjust the rod Q_2 and glass plate P_1 to obtain straight line fringes.

7. Mark a point of reference around the middle point of the P_2. Make the cursor coincide with a dark fringe near this point and note the microscope reading. This is the initial reading. Move the microscope cross-wire towards one side and note down the readings for every 5 fringe width up to 20 fringe width.
8. Switch ON the power supply connected to the heater. A voltmeter is connected across the heating filament for a coarse calibration of temperature. Usually a voltage reading of about 10-15 volt sets the temperature at 110-115^{0}C. So, initially to obtain a temperature rise of ~ 10-15^{0}C, vary the voltage from the power supply **very slowly** to get a reading of ~ 3-4V on the voltmeter. Wait for about 10 minutes till the temperature is stabilized. Note down the actual temperature displayed on the digital thermometer.
9. Now bring back the travelling microscope to the initial position and repeat step 7.
10. Continue the above step and record your readings for every 15°C interval (up to 115^{0}C).
11. Mark the point of contact of Q_2 and the glass plate P_1 using a marker pen. Using a scale measure the distance (l) between one edge (A) of the glass plate and the point (C) of contact.
12. Calculate the air-wedge angle 'θ' using Eq. 7 for all temperatures.

Observations:

Table 1: Measurement of L_{RT}

Least count of vernier calipers = ……. cm

S.No.	Reading at position A			Mean L_{RT} (cm)
	Main scale reading (cm)	Vernier scale reading (cm)	Total (cm)	

Table 2: Measurement of fringe width β:

λ = 589.3 nm Least count of travelling microscope = …..

Sl#	Temperature T (^{0}C)	No. of fringes	Microscope reading (cm)			Width of 5fringes	Fringe width	Average fringe width	Wedge angle
			Main scale	Vernier scale	Total				$\theta=\tan^{-1}(\lambda/2\beta)$

Graph: Plot a graph between air-wedge angle 'θ' vs. temperature 'T' and calculate the slope. Finally use Eq. 9 to calculate the coefficient of thermal expansion 'α'.

Results and Discussions

Error Analysis

Precautions:

1. Do not touch the heater or the rod by hand when the oven is ON.
2. Rotate the knob of power supply very slowly.
3. While adjusting to get the fringes, rotate the screw SC very gently, just enough to get the fringes.
4. Be careful while handling the glass plates.

References:

1. Born M., Wolf E., Principles of Optics.
2. Company manual
3. http://physics.info/expansion/ (for standard values of α)